

# New Security Results on Encrypted Key Exchange

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# Summary

- Contributions of this talk
  - Encrypted Key Exchange – example
  - Security Results
- One-Mask Diffie-Hellman Scheme
  - Password-based Authentication
  - Security Model
  - Analysis of the Protocol
  - Properties – Denial of service
- Conclusion

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# Key Exchange Schemes

- Alice and Bob agree on a common secret key  $sk$ , in order to establish a secret channel
- Intuitively: implicit authentication
  - only the intended partners can compute the session key
- Formally: semantic security
  - the session key  $sk$  is indistinguishable from a random string  $r$ , to anybody else

# Example: Diffie-Hellman

## ■ Diffie-Hellman Key Exchange

- $G = \langle g \rangle$ , cyclic group of prime order  $p$
- Alice chooses  $x \in \mathbb{Z}_p$  and sends  $X = g^x$
- Bob chooses  $y \in \mathbb{Z}_p$  and sends  $Y = g^y$
- Both can compute  $K = g^{xy}$

## ■ (Passive) Security under DDH Assumption

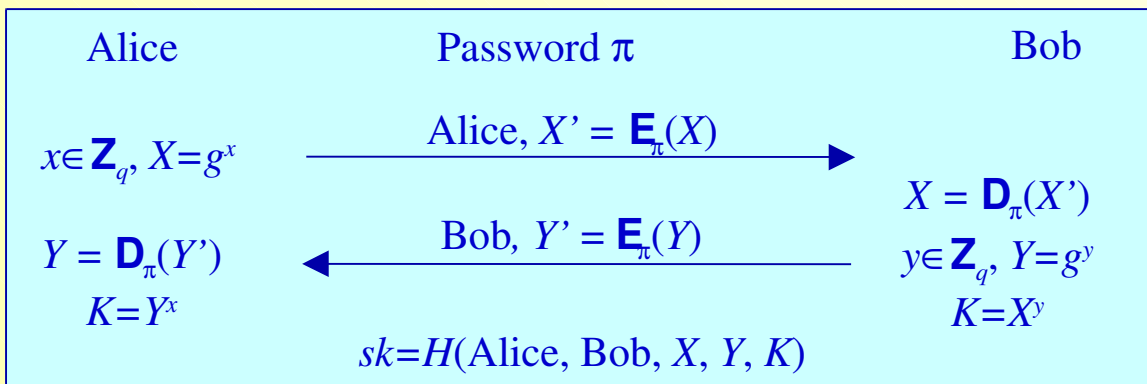
## ■ No security against active adversaries

- Authentication is needed

# How Authentication is Done

- **Asymmetric:**  $(sk_A, pk_A)$  and possibly  $(sk_B, pk_B)$ 
  - they authenticate to each other using the knowledge of the private key associated to the certified public key
- **Symmetric:** common (long / high-entropy) secret
  - they use the long term secret to derive a secure and authenticated ephemeral key  $sk$
- **Password:** common (short / low-entropy) secret
  - let us assume a 20-bit password

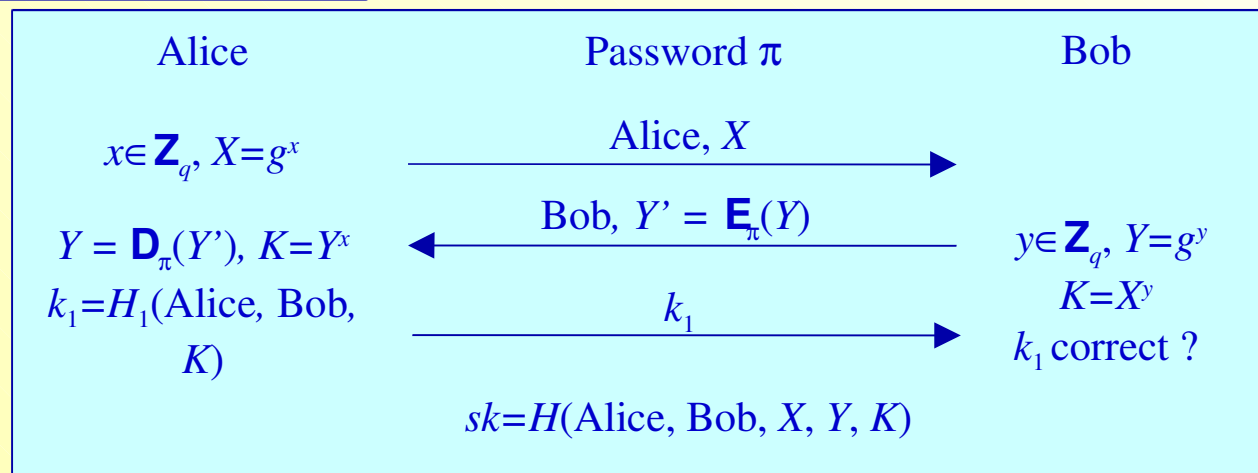
# EKE - AuthA



## EKE

Bellare-Merriott 1992

*Two-flow Encrypted Key Exchange*



## AuthA

Bellare-Rogaway 2000

*OEKE = One-flow Encrypted Key Exchange*

- Both schemes used an ``ideal cipher''

# New Results

- Provable security is achieved for both EKE and AuthA
  - In the random oracle model only
  - Based on CDH assumption
- Which means...
  - Security against dictionary attacks
  - Semantic security of the session key
- Add Denial-of-Service protection



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# Password-based Authentication

- Password (short / low-entropy secret – say 20 bits)
  - exhaustive search is possible
- Basic attack: on-line exhaustive search
  - the adversary guesses a password
  - tries to play the protocol with this guess
  - failure  $\Rightarrow$  it erases the password from the list
  - and restarts...
- after  $2^{20}$  attempts, the adversary wins

# Dictionary Attack

- The on-line exhaustive search
  - cannot be prevented
  - can be made less serious (delay, limitations, ...)

*We want it to be the best attack...*

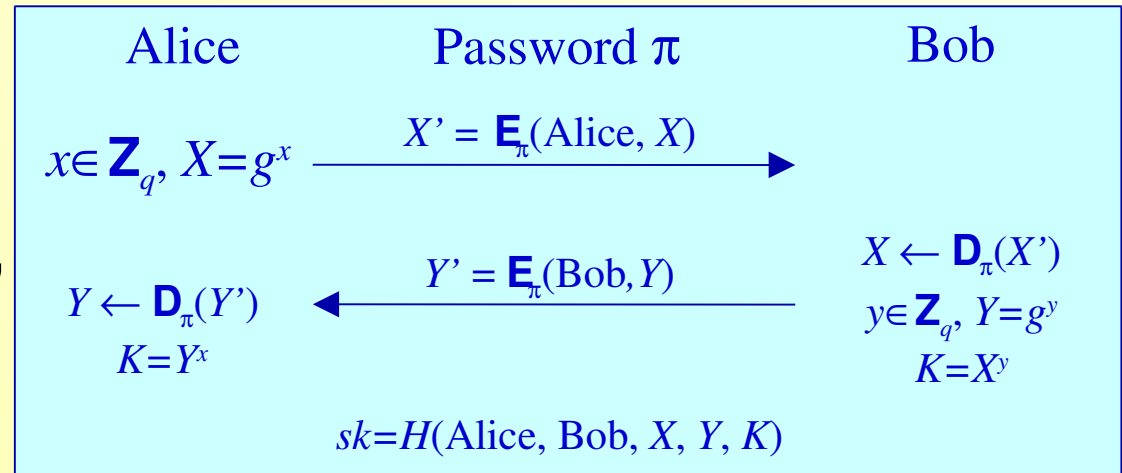
- The off-line exhaustive search
  - a few passive or active attacks
  - failure  $\Rightarrow$  erasure of MANY passwords from the list
  - this is called **dictionary attack**

# Example: EKE

- The most famous scheme EKE:  
Encrypted Key Exchange
- 2 flows are encrypted with the password.
- Must be done carefully: no redundancy
- For each password  $\pi$

***bad one !***

- decrypt  $X'$
- check whether  
it begins with “Alice”



# One-Mask Diffie-Hellman KE

Client A

Password  $\pi$  and  $\Pi=G(\pi)$

Server S

$x \in \mathbf{Z}_q, X=g^x$   $\xrightarrow{\text{Alice, } X^* = X \cdot \Pi}$

$X = X^* / \Pi$   
 $y \in \mathbf{Z}_q, Y=g^y$   
 $K=X^y$

$K=Y^x$   
 $\text{Auth}_S=?$   
 $H(A, S, X^*, Y, \Pi, K)$

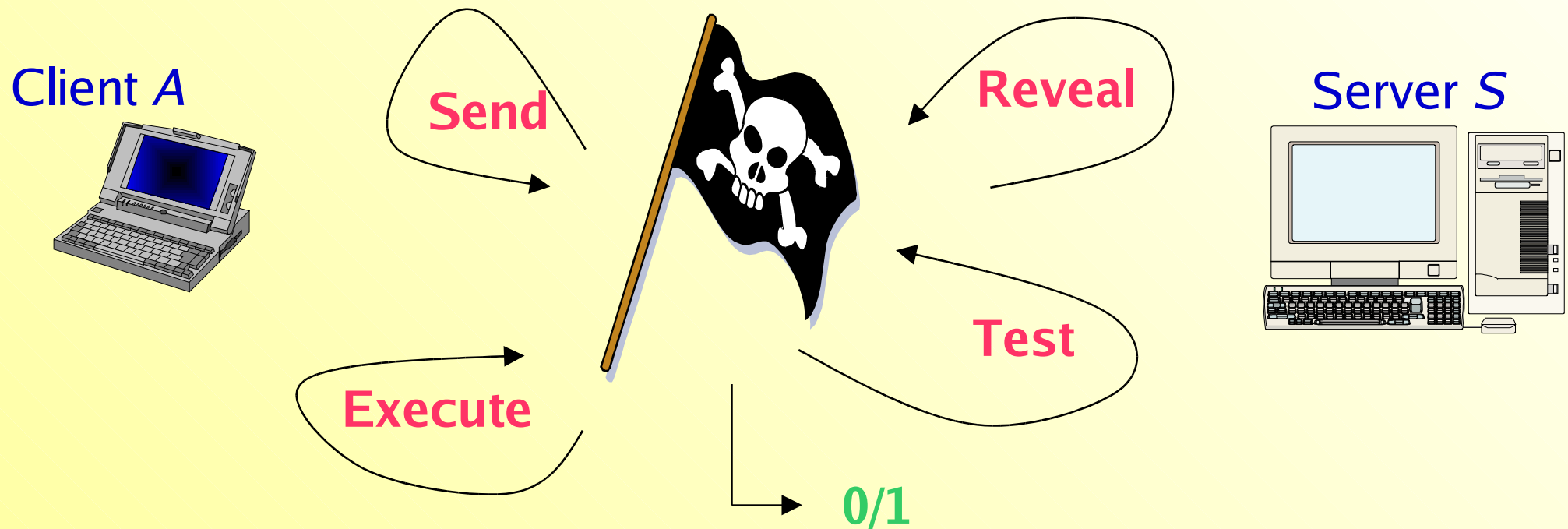
$\xleftarrow{\text{Bob, } Y, \text{Auth}_S}$

$\text{Auth}_S =$   
 $H(A, S, X^*, Y, \Pi, K)$

$Sk = H'(A, S, X^*, Y, \Pi, K)$

# Security Model

As many **Execute**, **Send** and **Reveal** queries as the adversary wants



But one **Test**-query, with  $b$  to be guessed...

# Passive/Active Adversaries

- Passive adversary: history built using
  - the **Execute**-queries  $\Rightarrow$  transcripts
  - the **Reveal**-queries  $\Rightarrow$  session keys
  - must learn *no information* about password
- Active adversary: entire control of the network
  - the **Send**-queries  $\Rightarrow$  send arbitrary messages
  - a **Send**-query allows to erase at most one password from the list

# Semantic Security

- For breaking the semantic security, the adversary asks one **Test**-query which is answered, according to a random bit  $b$ , by
  - the actual secret data  $sk$  (if  $b=0$ )
  - a random string  $r$  (if  $b=1$ )
- $\Rightarrow$  the adversary has to guess this bit  $b$



# OMDHKE: New Security Result

## ■ Assumptions

- the random-oracle model – for  $G$ ,  $H$  and  $H1$

## ■ Notations

- $q_s$ , the number of **Send**-queries (active and adaptive)
- $q_h$ , the number of **Hash**-queries to  $G$ ,  $H$  and  $H1$
- $N$ , the number of passwords

## ■ Semantic security of DHKE :

advantage  $\geq 12q_s/N + \epsilon$ ,

$\Rightarrow$  CDH problem : probability  $\geq \epsilon/qh^2$

(within almost the same time)

# One-Mask DHKE: the Proof

Client A

Password  $\pi$  and  $\Pi = G(\pi)$

Server S

$$x \in \mathbf{Z}_q, X^* = g^x \xrightarrow{\text{Alice, } X^* = X \cdot \Pi}$$

$$\begin{aligned} X &= X^* / \Pi \\ y \in \mathbf{Z}_q, Y &= g^y \\ K &= X^y \end{aligned}$$

$$\begin{aligned} &K = Y^x \\ \text{Auth}_S = ? &\xleftarrow{\text{Bob, } Y, \text{Auth}_S} \text{Auth}_S = \\ &H(A, S, X^*, Y, \Pi, K) \end{aligned}$$

$$sk = H_1(A, S, X^*, Y, \Pi, K)$$

# The Proof (2)

Client A

Password  $\pi$  and  $\Pi = G(\pi)$

Server S

$$x \in \mathbf{Z}_q, X^* = g^x \xrightarrow{\text{Alice, } X^* = X \cdot \Pi} \begin{array}{l} \text{Bob, } Y = g^y \\ \text{Auth}_S = H(A, S, X^*, Y, \Pi, K) \end{array}$$

~~$X = X^* / \Pi$~~   
 ~~$K = X^y$~~

$$\begin{array}{l} K = Y^x \\ \text{Auth}_S = ? \\ H(A, S, X^*, Y, \Pi, K) \end{array} \xleftarrow{\text{Bob, } Y, \text{Auth}_S} \begin{array}{l} \text{Auth}_S = \\ H(A, S, X^*, Y, \Pi, K) \end{array}$$

$$sk = H_1(A, S, X^*, Y, \Pi, K)$$

# The Proof (3)

Client A

Password  $\pi$  and  $\Pi = G(\pi)$

Server S

$$x \in \mathbf{Z}_q, \textcolor{red}{X^*} = g^x \xrightarrow{\text{Alice, } X^* = \textcolor{red}{X} \cdot \Pi} \begin{array}{l} \textcolor{red}{X} = X^* / \Pi \\ y \in \mathbf{Z}_q, Y = g^y \\ \textcolor{red}{K} = X^y \end{array}$$

$$\begin{array}{l} K = Y^x \\ \text{Auth}_S = ? \\ H(A, S, X^*, Y, \textcolor{red}{\Pi}, \textcolor{red}{K}) \end{array} \xleftarrow{\text{Bob, } Y, \text{Auth}_S} \begin{array}{l} \text{Auth}_S = \\ H(A, S, X^*, Y, \textcolor{red}{\Pi}, \textcolor{red}{K}) \end{array}$$

$$sk = H_1(A, S, X^*, Y, \textcolor{red}{\Pi}, \textcolor{red}{K})$$

# The Proof (4)

Client A

~~Password  $\pi$  and  $\Pi = G(\pi)$~~

Server S

$$x \in \mathbf{Z}_q, \textcolor{red}{X^*} = g^x$$

$$\text{Alice, } X^* = \textcolor{red}{X \cdot \Pi}$$

~~$$X = X^* / \Pi$$~~

$$y \in \mathbf{Z}_q, Y = g^y$$

~~$$K = X^y$$~~

$$K = Y^x$$

Bob,  $Y$ ,  $\text{Auth}_S$

$\text{Auth}_S =$

$\text{Auth}_S = ?$

$$H(A, S, X^*, Y, \textcolor{red}{\Pi}, \textcolor{red}{K})$$

$$H(A, S, X^*, Y, \textcolor{red}{\Pi}, \textcolor{red}{K})$$

$$sk = H_1(A, S, X^*, Y, \textcolor{red}{\Pi}, \textcolor{red}{K})$$

# One-Mask DH Key Exchange

- The simulated execution is indistinguishable from the real one, unless:
  - adversary asks the random oracle on values such as  $(A, S, X^*, Y, \Pi, K)$ 
    - if both  $X^*$  and  $Y$  are simulated from an instance of the DH problem, the adversary has solved it (when submitting  $K$ )
    - if one of these values is built by the adversary, it corresponds to an active attempt  $\Rightarrow$  at most  $q_s$
  - adversary has guessed the password by pure chance: proba  $\leq q_s / N$  since, the password is information-theoretically hidden in the simulation

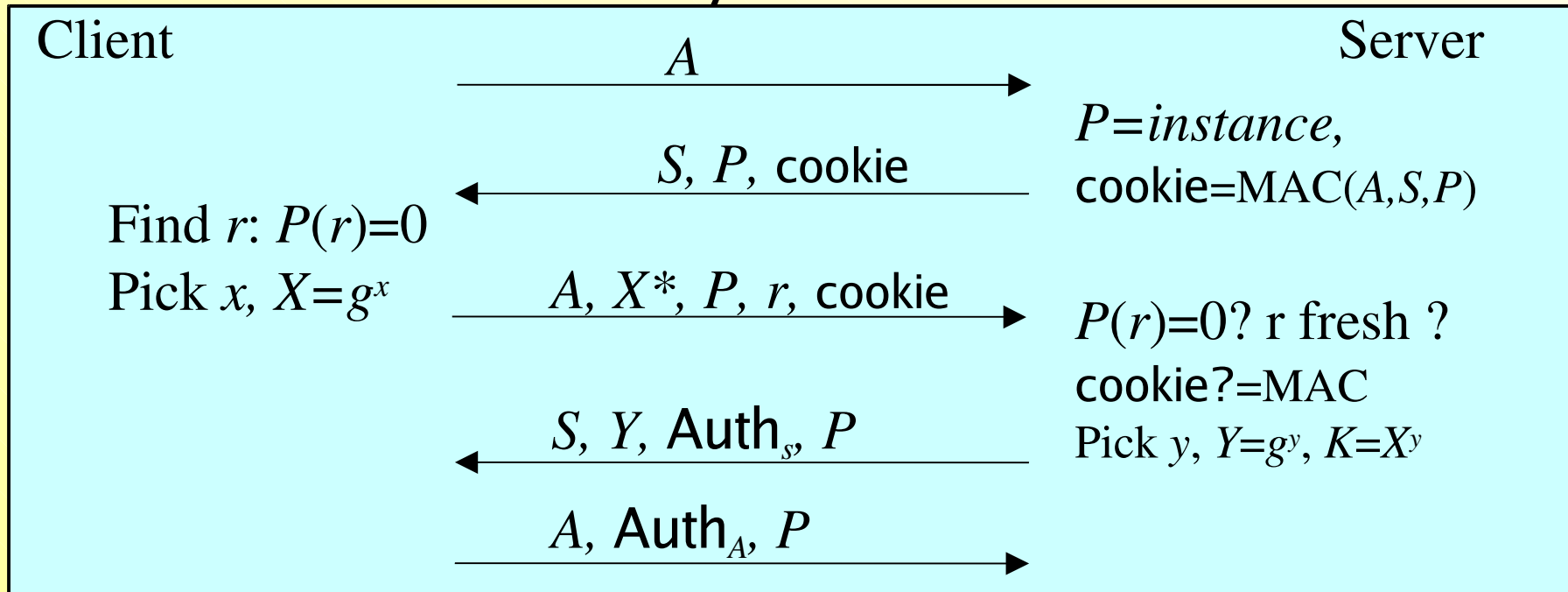
# DoS Resistance

## ■ Denial of service attacks

- The server never accepts anything, but rather crashes after memory exhaustion

## ■ Use of cryptographic puzzles

- Client has to perform a (small) exhaustive search
- Server can easily solve the correctness



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# Conclusion

- One-Mask and Two-Mask EKE variants are
  - provably secure in the random oracle model
  - semantic security
  - unilateral or mutual authentication
- More efficient than EKE
  - only one flow is encrypted
- More suitable for client-server schemes
  - the server can first send a generic flow not encrypted, and thus independent of the client